Conclusions

An analysis of the high-alpha lateral stability of 45-deg delta wings reveals that nonslender wing rock can only occur on a wing with rounded leading edges that trims at a large roll angle. In that case, the dipping, windward wing-half will experience dynamic undamping of the type observed on stalling airfoils, driving the wing-rock motion. In contrast, when the leading edge is sharp, no wing rock occurs even at high angles of attack and roll because the leeward surface generates a dead-air-type of flow that has little effect on the roll damping and the attached flow on the windward side generates sufficient roll damping to prevent the wing rock from developing.

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Calculating Flight Paths of Not Necessarily Small Inclination

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Introduction

ALTHOUGH the small flight-path angle approximation adequately treats routine operations of propeller/piston airplanes, its premise often fails to apply to modern jet aircraft with high thrust/weight ratios and might even be deficient for some maneuvers (e.g., steeply banked low-speed coordinated turns) of light airplanes. The historical reluctance[1] to move beyond the small path angle approximation was perhaps once understandable, but is no longer. This Note presents three graduated improvements (two analytical, one numerical) that can be used to solve a respectably comprehensive set (S-1) of aircraft center-of-mass equations of motion whenever better accuracy is needed. After describing those approximations and their solutions, sample calculations using each level of approximation are given for two jet aircraft climbing steeply at various bank angles. The author's future research plan is to use these enhanced flight-path approximations, together with improved modeling of the action of constant-speed propellers, to more realistically calculate tight or quick turn performance of Civil Air Patrol mountain search and rescue aircraft such as the Cessna 182 or 206.

In cases of steady banked flight, aircraft trajectories are portions of helices. A given helical flight path can be described by radius R of the (right circular) cylinder on which it is wound, angular rotation rate ω, and steady rate of climb or descent $\dot{h}$. It turns out, however, that neither is R the flight-path radius of curvature ρ nor is ω the airplane's angular speed $\dot{\chi}$. Exact relations between those two pairs of variables will be derived next.

Four Graduated Sets of Equations for Steady Aircraft Motion

The base set of equations we are concerned with, S-1, is

$$T\cos(\alpha+\alpha_T)-D-W\sin\gamma=0 \tag{1}$$

$$L\cos\varphi-W\cos\gamma+T\sin(\alpha+\alpha_T)=0 \tag{2}$$

$$L\sin\varphi-Wv^2/g\rho=0 \tag{3}$$

These equations come from considering forces 1) along the airplane's flight path, 2) perpendicular to the path and in the vertical direction, and 3) perpendicular to the path and in the horizontal direction. Those are directions of the common "trihedral" of right-handed orthonormal vectors $\hat{\boldsymbol{t}}$, $\hat{\boldsymbol{n}}$, $\hat{\boldsymbol{b}}$ of elementary differential geometry. Assuming the tie between lift L and angle of attack α given by a known relation $C_L(\alpha)$, Eqs. (1) and (2) are essentially in unknowns α (or lift L) and γ; Eq. (3) is in unknowns α (or L) and ρ.

Set S-1 incorporates assumptions that also apply (with further specific additions) to the three approximations to be considered: 1) no acceleration beyond the centripetal acceleration of a steady turn; 2) thrust offset α_T, if any, is constant and only in the airplane's plane of symmetry; 3) no side forces (only coordinated flight); 4) no "kinetic energy effect"; 5) aircraft weight W is constant during any single maneuver, as are thrust T, scalar airspeed v, and all of the rest of the featured variables; and 6) the drag polar is quadratic, $C_D=C_{D0}+KC_L^2$. The remainder of this section presents the four equation sets, in increasing order of exactitude, and remarks on their solutions. Frequent use will be made of standard variables and relations, such as

$$q\equiv\tfrac{1}{2}\rho v^2,\qquad L(\alpha)=qSC_L(\alpha)$$

$$D(\alpha)=D_P+D_i(\alpha)=qS\left[C_{D0}+KC_L^2(\alpha)\right] \tag{4}$$

often with additional subscripts to indicate the approximation being considered.

Small Flight-Path Angle γ_s (and Small Angles of Attack), S-4

This approximation is zeroth order in angles of attack—$\cos(\alpha+\alpha_T)\doteq 1$ and $\sin(\alpha+\alpha_T)\doteq 0$ and treats angle γ as small in the sense that $\cos\gamma\doteq 1$ but $\sin\gamma$ remains intact. The effect is to disentangle lift L from γ and immediately give one of the three needed solutions as

$$L_s=W/\cos\varphi \tag{5}$$

(Subscript s is for small flight-path angle.) Using the second of Eqs. (4), one gets

$$D_s=qSC_{D0}+\frac{KW^2}{qS\cos^2\varphi}=D_P+D_{\rm is} \tag{6}$$

Because Eq. (1) is now approximated as

$$T-D_s-W\sin\gamma_s=0 \tag{7}$$

we have a second and more important solution as

$$\sin\gamma_s=(T-D_s)/W \tag{8}$$

where it is understood that $(T-D_s)\ll W$. It is only left to solve Eq. (3), understanding a somewhat deficient $L\approx L_s$, to get (just as for level steady coordinated turns)

$$\rho_s=\frac{Wv^2}{gL_s\sin\varphi}=\frac{v^2}{g\tan\varphi} \tag{9}$$

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Angle γ_n Not Necessarily Small (and Small Angles of Attack), S-3

This approximation too is zeroth order in the angles of attack, but it leaves both appearances of path angle intact. The equations of motion are

$$T - D_n - W \sin\gamma_n = 0 \tag{10}$$

$$L_n \cos\varphi - W \cos\gamma_n = 0 \tag{11}$$

$$L_n \sin\varphi - W v^2/g\rho = 0 \tag{12}$$

(Subscript n stands for "not necessarily small.") Equation (11) is solved for lift to give

$$L_n = W \cos\gamma_n / \cos\varphi \tag{13}$$

(as yet only an incipient solution because γ_n is still unknown). Then of course

$$C_{\mathrm{Ln}}(\alpha) = \frac{L_n}{qS} = \frac{W\cos\gamma_n}{qS\cos\varphi} \tag{14}$$

and

$$D_n = qSC_{D0} + \frac{KW^2\cos^2\gamma_n}{qS\cos^2\varphi} = D_P + D_{\mathrm{in}}$$

$$= D_P + D_{\mathrm{is}}\left(1 - \sin^2\gamma_n\right) \tag{15}$$

which motivates the punctilious use of subscripts. With Eq. (15) substituted into it, Eq. (10) becomes a quadratic in $\sin\gamma_n$—standard form $Ax^2 + Bx + C = 0$, with solutions written as simple corrections to the small flight-path angle approximation $\sin\gamma_s$. That quadratic is

$$(D_{\mathrm{is}}/W)\sin^2\gamma_n - \sin\gamma_n + \sin\gamma_s = 0 \tag{16}$$

with coefficients readily available from the small flight-path angle approximation. Here the general quadratic solutions

$$x = \frac{2C}{-B \mp \sqrt{B^2 - 4AC}} \tag{17}$$

give specific solutions

$$\sin\gamma_n = \frac{2\sin\gamma_s}{1 \mp \sqrt{1 - 4D_{\mathrm{is}}\sin\gamma_s/W}} \tag{18}$$

In most (but not all) cases only the lower sign gives physically realizable results. With γ_n in hand, L_n is available from Eq. (13), and, using it, the radius of curvature is

$$\rho_n = \frac{Wv^2}{gL_n\sin\varphi} = \frac{v^2}{g\tan\varphi\cos\gamma_n} \tag{19}$$

Low Aerodynamic Angles of Attack α (with $\alpha_T = 0$), S-2

This approximation, besides forcing $\alpha_T = 0$, is second order in trigonometric functions of α and also restricts $C_L(\alpha) = a\alpha$ to its linear portion. Equations (1) and (2) become

$$T(1 - \alpha^2/2) - D_2 - W\sin\gamma_2 = 0 \tag{20}$$

$$L_2\cos\varphi - W\cos\gamma_2 + T\alpha = 0 \tag{21}$$

Equation (3) is unchanged, although through these approximations $L_2 < L$ very slightly. Using $L_2 = qSC_L(\alpha) = qSa\alpha$, Eq. (21) is linear in α. Substituting the resulting expression

$$\alpha = \frac{W\cos\gamma_2}{T + qSa\cos\varphi} \tag{22}$$

into Eq. (20), one gets a quadratic in $\sin\gamma_2$ with standard coefficients:

$$A = W\frac{T/2 + qSKa^2}{(T + qSa\cos\varphi)^2}, \qquad B = -1$$

$$C = \frac{T - qSC_{D0}}{W} - A \tag{23}$$

The solutions are then

$$\sin\gamma_2 = \frac{2C}{1 \mp \sqrt{1 - 4AC}} \tag{24}$$

$$\rho = v^2\frac{1 + T/qSa\cos\varphi}{g\tan\varphi\cos\gamma_2} \tag{25}$$

$$L_2 = \frac{qSaW\cos\gamma_2}{T + qSa\cos\varphi} \tag{26}$$

As will be seen in numerical examples, these implied analytic solutions to S-2 give results very close to exact numerical solutions to S-1. Unstalled angles of attack are more restricted to small than are modern-day flight-path angles.

A loosened version of S-2, without requiring $\alpha_T = 0$, could also be pushed through analytically. Then Eqs. (1) and (2) become

$$T\left[1 - (\alpha + \alpha_T)^2/2\right] - D - W\sin\gamma = 0 \tag{27}$$

$$L\cos\varphi - W\cos\gamma + T(\alpha + \alpha_T) = 0 \tag{28}$$

One first gets an expression for $(\alpha + \alpha_T)$ from Eq. (28)—treating α as $[(\alpha + \alpha_T) - \alpha_T]$—and substitutes that expression into Eq. (27) to get terms in 1, $\sin\gamma$, $\cos\gamma$, and $\cos^2\gamma$. One then changes variable to $\tan\gamma$ and ends with a quartic with complicated coefficients. In view of the close approximations afforded by solutions to S-2, or by the not difficult numerical solutions of S-1, solving that quartic does not appear worthwhile.

Unrestricted Base Set of Equations of Motion, S-1, and Their Numerical Solution

Although there is no general (nonspecial case) analytic solution to Eqs. (1–3), they can be solved numerically:

Step 1) Isolate the single terms in flight-path angle γ in each of Eqs. (1) and (2), square both expressions, and add them. Angle γ is thereby eliminated. One obtains:

$$W^2 - T^2 - L^2(\alpha)\cos^2\varphi - D^2(\alpha) + 2TD(\alpha)\cos(\alpha + \alpha_T)$$

$$- 2TL(\alpha)\cos\varphi\sin(\alpha + \alpha_T) = 0 \tag{29}$$

Step 2) Use the latter two of Eqs. (4) and squares of both those expressions in Eq. (29). One gets the following expression, terms organized according to factors featuring unknown α:

$$\left[W^2 - T^2 - (qSC_{D0})^2\right] + \cos(\alpha + \alpha_T)2TqS\left[C_{D0} + KC_L^2(\alpha)\right]$$

$$- \sin(\alpha + \alpha_T)2TqSC_L(\alpha)\cos\varphi - [qSC_L(\alpha)]^2$$

$$\times\left(\cos^2\varphi + 2KC_{D0}\right) - \left[qSKC_L^2(\alpha)\right]^2 = 0 \tag{30}$$

Step 3) Solve Eq. (30), numerically, for α (in radians). (The Excel spreadsheet program Solver facility is sufficient.)

Step 4) With α now known, numerically solve Eq. (1) [and Eq. (2), as a check] for γ.

Step 5) Calculate lift L by using the second of Eqs. (4).

Step 6) Find the path radius of curvature by rearranging Eq. (3), using known L, to get

$$\rho = Wv^2/gL\sin\varphi \tag{31}$$

Table 1 Sample aircraft parameters and operating data

Data item	F104-G	F-16C
C_{D0}	0.018	0.018
K	0.20	0.1326
a, rad^{-1}	2.85	3.77
S, ft^2	196	300
T, lbf	15,000	20,000
W, lbf	18,000	23,000

Table 2 Flight-path data from four sets of equations of motion

Aircraft	F104-G			F-16C		
Airspeed, ft/s	422	464	591	295	295	295
Bank angle, deg	0	30	60	0	30	60
S-4						
γ_s, deg	44.8	43.4	35.1	48.3	45.6	26.9
L_s, lbf	18,000	20,785	36,000	23,000	26,558	46,000
ρ_s, ft	NA	11,598	6,262	NA	4,696	1,566
S-3						
γ_n, deg	49.0	47.7	40.5	54.3	52.9	36.1
L_n, lbf	11,815	13,984	27,371	14,431	16,017	37,172
ρ_n, ft	NA	17,238	8,236	NA	7,787	1,937
S-2						
γ_2, deg	49.4	48.3	42.2	54.8	53.9	46.4
L_2, lbf	10,386	12,340	23,621	11,318	13,068	23,675
ρ_2, ft	NA	19,534	9,544	NA	9,545	3,042
S-1						
γ_1, deg	49.4	48.3	42.2	54.8	53.9	46.3
L_1, lbf	10,388	12,342	23,627	11,321	13,073	23,737
ρ_1, ft	NA	19,531	9,542	NA	9,541	3,034

Sample Numerical Solution Results

Two jet fighters, both at mean sea level (MSL) with flaps up, were chosen: 1) F104-G, aircraft parameters from Adamson,[2] and 2) F-16C, parameters from Asselin.[3] Details are in Table 1. Airspeeds v were picked arbitrarily but always above the relevant stall speed.

Table 2 displays solutions of the four sets of equations of motion for these two aircraft under the cited conditions, at MSL, flaps up, $\alpha_T = 0$.

Helical Flight Paths

When banked, the airplane's trajectory is a portion of a helix. To prove that fact, one can integrate approximate equations of motion either in cylindrical coordinates (carefully) or in Cartesian ones. Or, one can take a specimen helical path, parameterized by R, ω, $\dot{h} = v \sin\gamma$, and show that a mass following that path at constant speed v must be acted on by forces mirroring the equations of motion. But in fact one has no doubt that steady banked flight results in helical flight paths. The question is, Which helix?

One clue comes from the fact that the airplane's horizontal component of velocity, $v\cos\gamma$, must equal $R\omega$. In addition, we know from dynamics that $v = \rho\dot{\chi}$, and so $\rho\dot{\chi}\cos\gamma = R\omega$. One further relation is needed. Consider a coordinate system O^* parallel to our usual Cartesian system O and moving uniformly in the Z direction at speed $\dot{h}$. From the point of view of O^*, the airplane is simply moving with speed $R\omega = v\cos\gamma$ in a horizontal circle of radius R; hence, it must have force $\boldsymbol{F}^* = [m(R\omega)^2/R]\hat{\boldsymbol{n}} = [m(v\cos\gamma)^2/R]\hat{\boldsymbol{n}}$ acting on it. But because O^* is not accelerated with respect to O, $\boldsymbol{F}^*$ must equal the force as seen from the O system, $\boldsymbol{F} = (mv^2/\rho)\hat{\boldsymbol{n}}$. Hence,

$$\rho = R/\cos^2\gamma \tag{32}$$

Then from the earlier relation one finds

$$\dot{\chi} = \omega\cos\gamma \tag{33}$$

so that always $\rho > R$ and $\dot{\chi} < \omega$, as makes intuitive sense.

Conclusions

In a specific case in which one questions validity of the small flight-path angle approximation (set S-4), several analytical or numerical procedures stand ready to settle that question and, if necessary, to supplant that inadequate approximation. Set S-3 gave markedly better results than S-4 with very little additional effort. Set S-2, though yielding a more complicated quadratic, gave results almost as good as the exact numerical solutions to set S-1. Once set up, even that last procedure takes only a few minutes. The confusing relations between radii of curvature and angular speeds, looked at from the alternative aircraft dynamics and helix kinematics points of view, were clarified.

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Radar Cross Section Constraints in Flight-Path Optimization

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Introduction

THE Department of Aeronautics at the Royal Institute of Technology has for some time been involved in developing methods for aircraft trajectory optimization. The optimized trajectories have been flight tested by the Swedish Air Force using the supersonic Saab J35 Draken and the jet trainer Saab 105 (Ref. 1–4).

Radar is the only threat against aircraft considered in this study. The detection time is defined as the time interval between the instant at which the aircraft is first detected and the instant at which the aircraft reaches the specified target. The detection distance is the distance from the target to the position at which the aircraft is first detected by radar. Given an initial aircraft position and a target position, the offset distance is defined as the perpendicular distance to an alternative flight path parallel to the original flight path. Hence, a flight path pointing directly at, or above, the target is defined to have zero offset.

In a previous study substantial decrease in detection time was experienced and verified in flight tests. This was achieved without any optimization methods applied.[5] The purpose of the present study is to develop a radar cross section (RCS) constraint suitable for three-dimensional flight-path optimization. To be computationally efficient, such an RCS representation has to be continuous and differentiable. To gain understanding of the potential decrease in detection time, numerical examples are considered.

Performance Model

Flight-path optimization is often performed in two dimensions, only considering the longitudinal degrees of freedom. Such a model is not suitable when RCS properties are considered because the RCS can fluctuate significantly even for small changes in pitch and bank angles.[6] The full-blown six-degree-of-freedom (6-DOF) model is

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